(DM 03)

M.Sc. Mathematics (Previous) DEGREE EXAMINATION, MAY 2006

Paper - III : COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL DIFFERENCIAL EQUATIONS

Time : Three hours   Maximum : 100 marks

Answer any FIVE choosing atleast TWO questions from each Part.

PART - A

1. (a) When n is a postive integer, prove that

$$P_n(x) = \frac{1}{\pi}\int_0^{\pi} \frac{d\phi}{[x \pm \sqrt{x^2-1}\cos\phi]^{n+1}}$$

(b) Prove that $(1-x^2)P'_n = (n+1)(xP_n - P_{n-1})$

2. (a) Prove that

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$$

(b) Prove that

$$\int_{-1}^{1}(x^2-1)P_{n+1}P_n' dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

3. (a) State and prove the generating function for $J_n(x)$.

(b) Prove that

$$\int_0^{\infty} e^{-ax}J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}, a>0$$

4. (a) State and prove the necessary and sufficient condition for integrability $Pdx + Qdy + Rdz = 0$

(b) Show that there is no single integral of $dz = 2ydx + xdy$. Prove that the curves of this equation that lie in the plane $z = x + y$ lie also on the surface of the family $(x-1)^2(2y-1) = C$.

5. (a) Apply Charpits method to find the complete integral of $z^2(p^2z^2+q^2) = 1$.

(b) Solve $r - t\cos^2 x + p\tan x = 0$ by using Monge's method.

PART - B

6. (a) Derive the correspondence between the points of unit sphere S and C_∞.

(b) Let $z = cis\frac{2\pi}{n}$ for an integer $n \geq 2$. Show that $1 + z + ... + z^{n-1} = 0$.

7. (a) If $\sum a_n(z-\alpha)^2$ is a given power series with radius of convergence R, then prove that

$R = \lim\left|\frac{a_n}{a_{n+1}}\right|$ if this limit exists.

(b) Let G be either the whole plane C or some open disk. If $u : G \to \mathbf{R}$ is hormonic function, then prove that *u* has a hormonic conjugate.

8. (a) If $\gamma : [a, b] \to C$ is Piecewise smooth, then prove that γ is of bounded variation and

$$V(\gamma) = \int_a^b |\gamma'(t)| dt.$$

(b) State and prove the fundamental theorem of calculus for line integrals.

9. (a) Evaluate $\int_\gamma \frac{z^2+1}{z(z^2+4)} dz$ where $\gamma(t) = \gamma e^{it}$, $0 \le t \le 2\pi$ for all positive values of $\gamma, 0 < \gamma < 2$ and $2 < \gamma < \infty$.

(b) State and prove Morera's theorem.

10. (a) Show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

(b) State and prove Rouche's theorem.

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